Modeling of Dependence in Impulsive Interference and Copula Theory

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ET TECHNOLOGIES

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1. Modeling of Interference
2. Copula Theory
3. System Model
4. Simulations

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Modeling of Interference

Interference Model

I. Classic Gaussian Model:

1) Impulsiveness ---- low tail probability
2) Dependence --- i.i.d

II. Modeling of Impulsiveness

Alpha-stable Model --- long tail in PDF

III. Modeling of Dependence:

1) Sub-Gaussian Model
2) Copula Theory

Modeling of Interference

Alpha-stable model:

Network Geometry --- PPP (Poisson Point Process)

Aggregate interference --- α-Stable[1].

1) Characterizes impulsivness
2) Captures the characteristics of network geometry

[1] ILOW, Jacek et HATZINAKOS, Dimitrios. Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers. *IEEE transactions on signal processing*, 1998, vol. 46, no 6, p. 1601-1611..

Modeling of Interference

Dependence of interference in wireless network

Two determinants of interference[2]**:**

1. Path Law, e.g. SIMO – strong interference received on one antenna → strong one on another antenna.

2. **Network Geometry**, (concurrently transmitting nodes).

(1) the distribution of nodes

(2) the multiple access scheme. E.g. SCMA

[2] Weber, Steven, and Jeffrey G. Andrews. "Transmission capacity of wireless networks." *Foundations and Trends® in Networking*5.2–3 (2012): 109-281.

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Copula Theory

Question : What is copula?

1. Sklar's Theorem:

If $F\,(X_1, \ldots, Xn)$ is joint distributed function with margins $F_1(X_1), \ldots, F_n(X_n)$, there exists a (Copula) funcion C such that

$$F\left(X_1, \ldots, X_n\right) = C\left(F_1\left(X_1\right), \ldots, F_n\left(X_n\right)\right) \qquad (2\text{-}1)$$

where C is unique if F and G are continuous, otherwise C is uniquely determined on $\mathrm{Ran}(F_1) \times \cdots \times \mathrm{Ran}(F_n)$.

$$C\left(U_1, \ldots, U_n\right) = F\left(F_1^{-1}\left(U_1\right), \ldots, F_n^{-n}\left(U_n\right)\right) \qquad (2\text{-}2)$$

Copula Theory

Question : What is copula?

2. PDF of $\vec{X} = (X_1, \ldots, Xn)$:

$$f(X_1, \ldots, X_n) = \underbrace{c\left(f(X_1), \ldots, f(X_n)\right)}_{\text{Dependent component}} \prod_{k=1}^{n} f(X_k) \tag{2-3}$$

Independent component

$$c(u_1, \ldots, u_n) = \frac{\partial^n C(u_1, \ldots, u_n)}{\partial u_1 \ldots \partial u_n} \tag{2-4}$$

Copula Theory

Question : why copula?

3. Receiver:

In a BPSK system, the received signal is:

$$\vec{Y}_N = \vec{S}_N + \vec{Z}_N \tag{2-5}$$

$\vec{S}_N$ ---- vector containing repeated samples $s = \pm 1$,

$\vec{Z}_N$ ---- interference vector.

Log likelihood ratio (**LLR**) will be:

$$\wedge\left(\vec{Y}_N\right) = \log \frac{P\left(z_1 = y_1 - 1, \ldots, z_N = y_N - 1 \mid s = +1\right)}{P\left(z_1 = y_1 + 1, \ldots, z_N = y_N - 1 \mid s = -1\right)}$$

$$= \log \frac{f_{\vec{Z}_N}\left(y_1 - 1, \ldots, y_N - 1\right)}{f_{\vec{Z}_N}\left(y_1 + 1, \ldots, y_N + 1\right)} \tag{2-6}$$

where f is the joint **PDF** of $\vec{Z}_K$.

Copula Theory

Question : Relation between copula and our research

3. Receiver:

Hence, the LLR becomes:

$$\underbrace{\wedge\left(\vec{Y}_N\right)=\log\frac{c\left(F\left(y_1-1\right),\ldots,F\left(y_N-1\right)\right)}{c\left(F\left(y_1+1\right),\ldots,F\left(y_N+1\right)\right)}}_{\wedge_c(Y)}+\underbrace{\sum_{i=1}^{N}\log\frac{f\left(y_i-1\right)}{f\left(y_i+1\right)}}_{\wedge_\perp(Y)} \quad (2\text{-}7)$$

Dependent strcuture

Independent structure

tricky

Copula Theory

4. Student (*t*) copula:

$$C_{v,\Sigma}\left(u_1,\ldots,u_n\right)=F\left(F_1^{-1}\left(u_1\right),\ldots,F_1^{-n}\left(u_n\right)\right) \tag{2-8}$$

where $F \sim T_n(0,\Sigma,v)$ and $F_1 \sim T(v)$

$$c_{v,\Sigma}\left(\vec{u}\right)=\frac{f\left(t^{-1}\left(u_1\right),\ldots,t^{-1}\left(u_n\right)\right)}{\prod_{i=1}^{n} f_1\left(t^{-1}\left(u_i\right)\right)} \tag{2-9}$$

where f and f_1 are densities for *n* and 1 dimension

1. Flexibility and wide range of dependence -- v , Σ
2. Extendable
3. Closed-form or computationally feasible,

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System Model

Model (PPP with λ) :

1. Each device transmits over a subset of orthogonal bands $B = \{1, \dots, K\}$
2. Each time *t*, devices independently transmit on a band $k \in B$ with probability p
3. Devices on band k --- thinning PPP wih $p\lambda$

$$Z_k(t) = \sum_{j \in \Phi_k(t)} r_j(t)^{-\eta/2} h_{j,k}(t) x_{j,k}(t), \quad k = 1, \dots, K \tag{3-1}$$

System Model

Interference is

$$Z_k(t) = \sum_{j \in \Phi_k(t)} r_j(t)^{-\eta/2} h_{j,k}(t) x_{j,k}(t), \quad k = 1, \ldots, K \qquad \text{(3-1)}$$

$h_{j,k}(t) \sim CN(0,1)$ ---- Rayleigh fading of device *j* on band *k*
$r_j(t)$ ---- distance to the access point
η ---- path-loss exponent
$\Phi_k(t)$ ---- PPP with $p\lambda$

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System Model

1. $Z_k = z_{k,r} + i \cdot z_{k,i}$ is an isotropic complex α-stable variable, where α=4/η.

$$\vec{Z}_k = \left(z_{k,r}, z_{k,i}\right) = A^{1/2}\left(G_1, G_2\right) \qquad (3\text{-}2)$$

$\vec{Z}_k = A^{1/2}\left(G_1, G_2\right)$ is sub-Gaussian, where

$A \sim S_{\alpha/2}\left(\cos\left(\frac{\pi}{4}\alpha\right)^{2/\alpha}, 1, 0\right)$ ---- skewed α-stable

G_1, G_2 ---- independent Gaussian variables, i.e

$\left(G_1, G_2\right) \sim CN(0, \sigma^2 I)$

System Model

Scenario --- General SCMA codebook

2. Stacking real and imaginary parts, we have the *2k* dimensional interference vector

$$\vec{Z}_{2K} = \left(z_{1,r}, z_{1,i}, \ldots, z_{K,r}, z_{K,i} \right) \qquad (2\text{-}3)$$

Sub–Gaussian for p=1.

Dependence structure remains unspecified (0<p<1).

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Simulation

Each of these devices (following PPP) transmits on each band k=1,…,K with probability p.

pK ---- the average quantity of frequency resources used by each device per transmission .

Simulation

1. Dependence between Real and Imaginary parts: $\vec{Z}_k = (z_{k,r}, z_{k,i}) = A^{1/2}(G_1, G_2)$

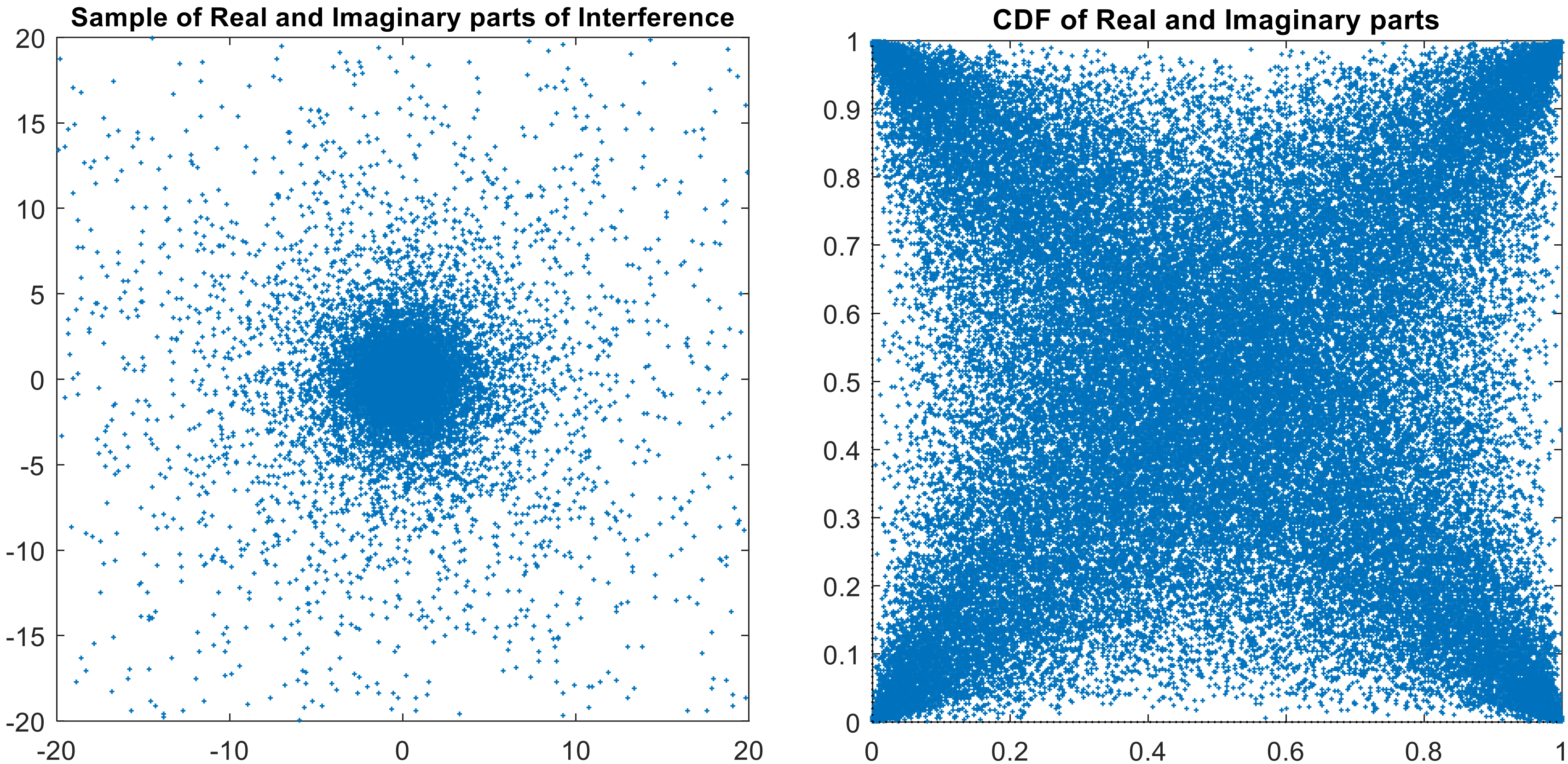

Figure 1. Real and Imaginary Parts of Interference

1. Isotropic
2. Conditionally Gaussian
3. Strong dependence in impulsiveness

Simulation 2. Dependence between Real parts:

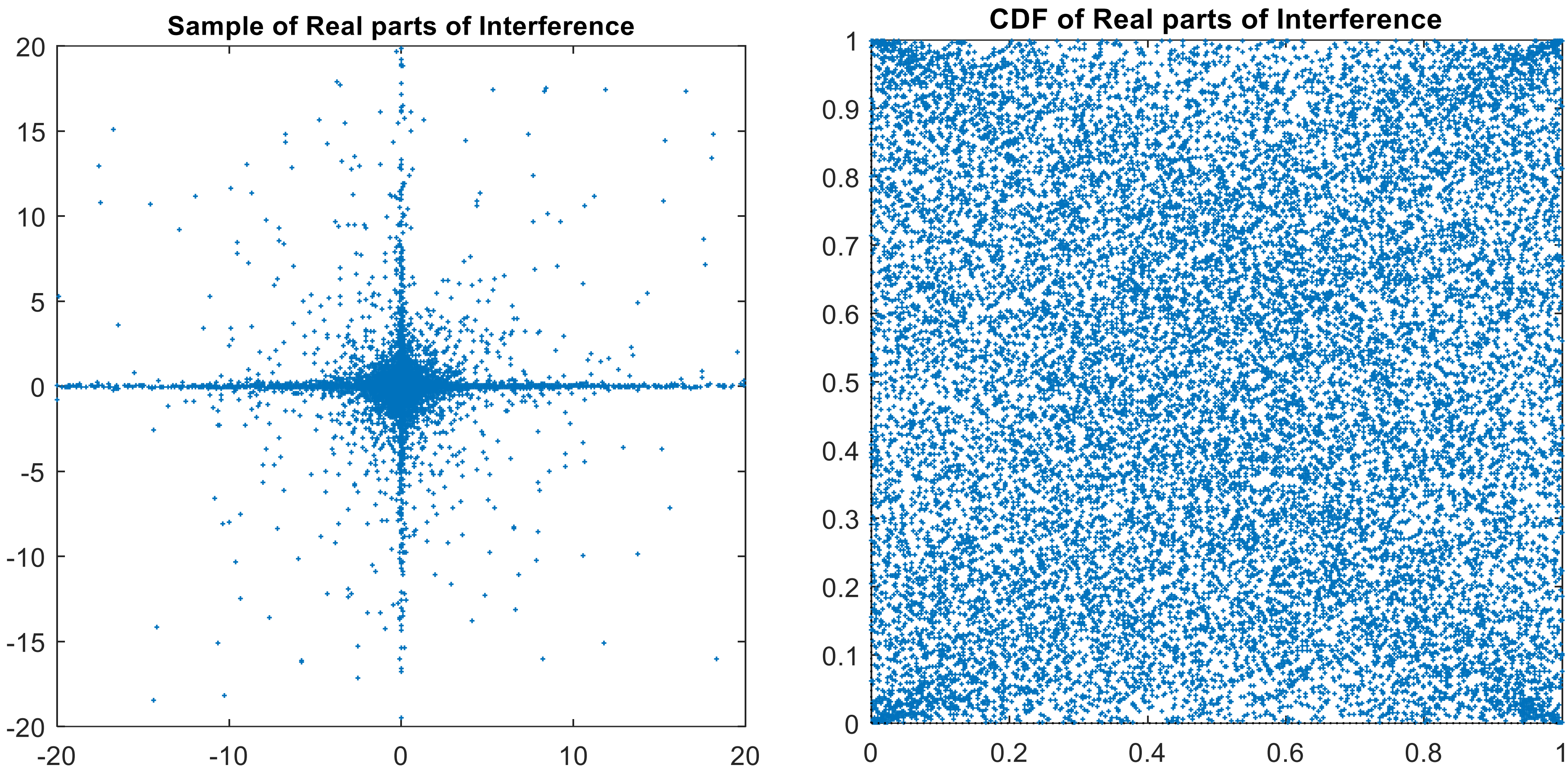

Figure 2. Real Parts of Interference on different bands

$$\vec{Z}_k = \left(z_{1,r}, z_{2,r}\right)$$

Simulation

3. Density of its margins(CDF):

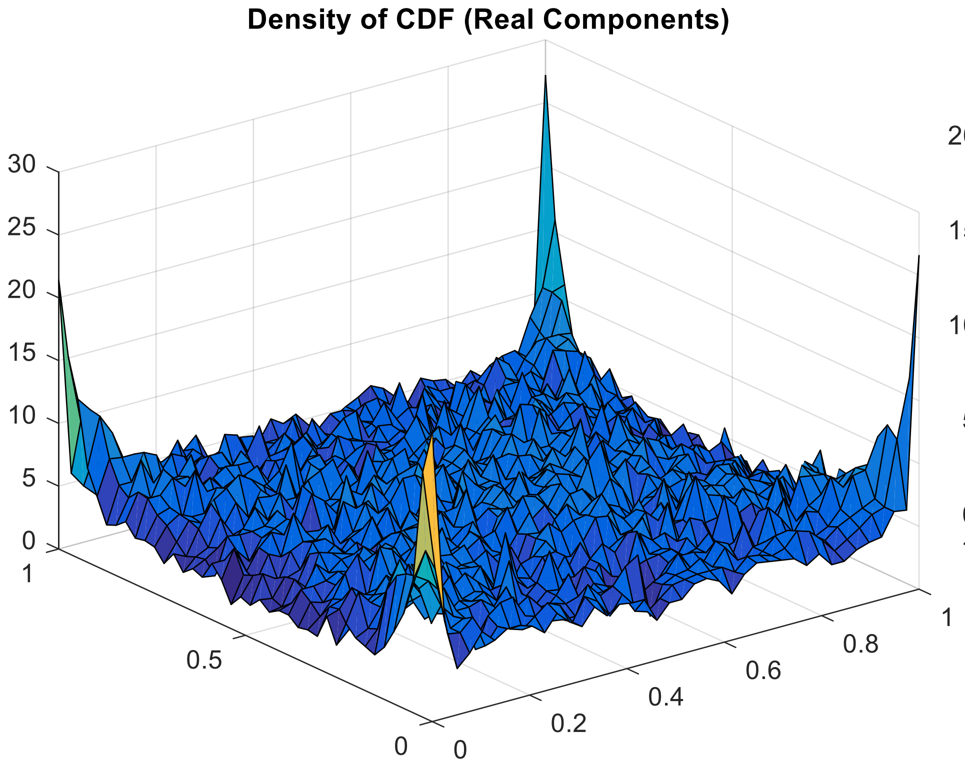

Figure 3. Density of margins of α-stable interference

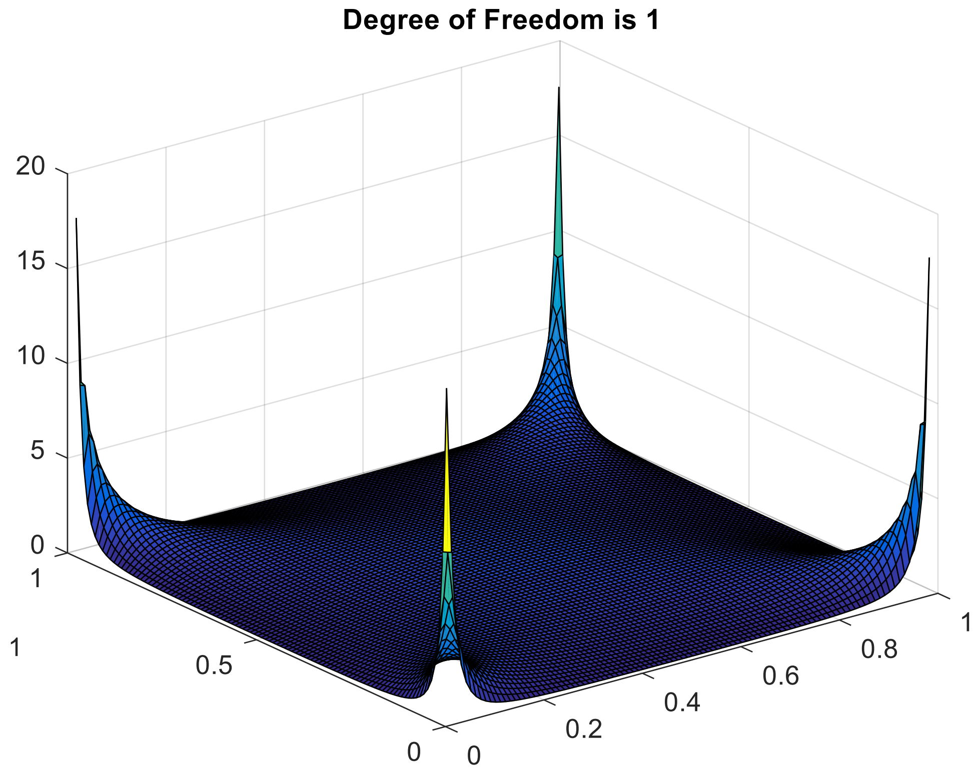

Figure 4. Density of *t* copula function

T copulas with different *v* have similar densities.

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Simulation

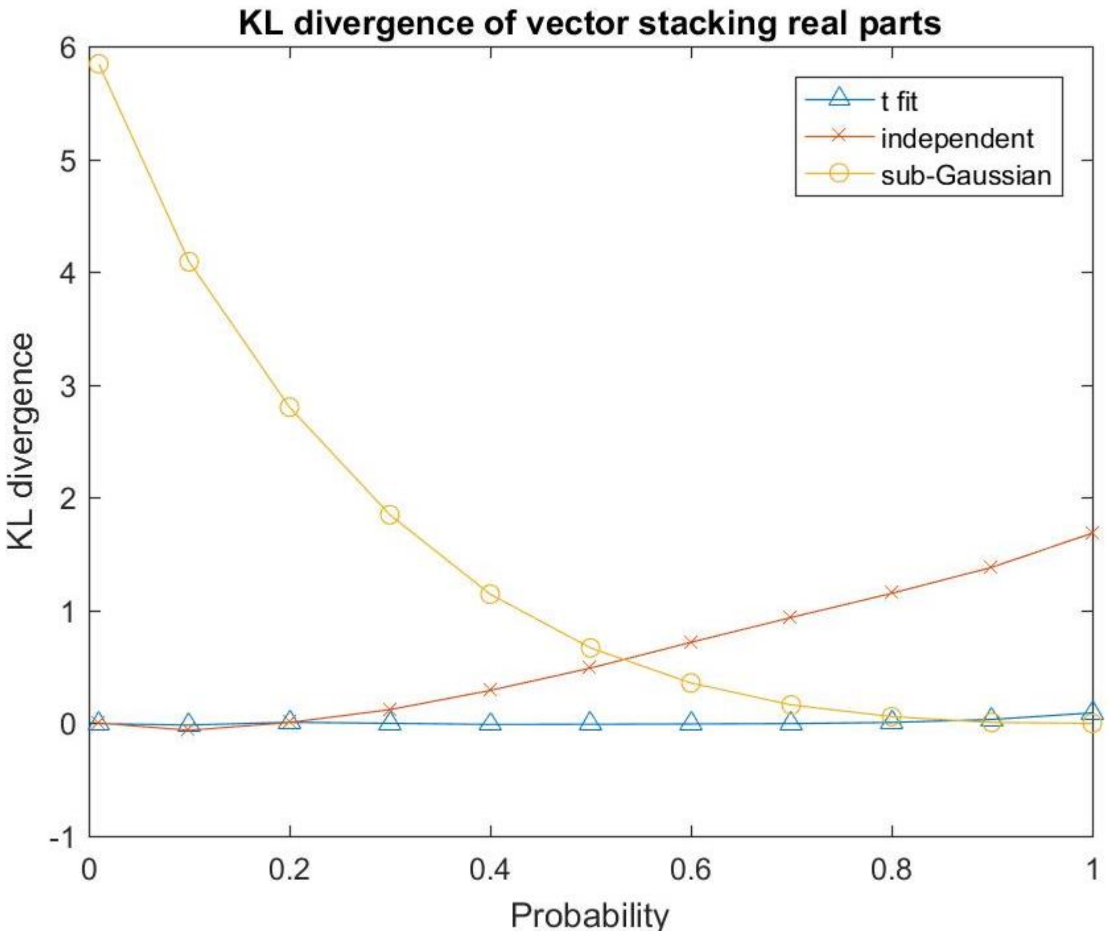
1. Monte Carlo Method --- (Test) Data set
2. Stable fit & T Copula fit --- Estimating Model Parameters
3. Generating datas under the model with the estimated parameters --- (Model) Data Set
4. KL-divergence between **Test Set** and **Model Set**.

Three models are compared:

1. t copula
2. 2-dimensional sub=Gaussian
3. Sub-Gaussian

$$\vec{Z}_{2K} = \left(z_{1,r}, z_{1,i}, \ldots, z_{K,r}, z_{K,i} \right)$$

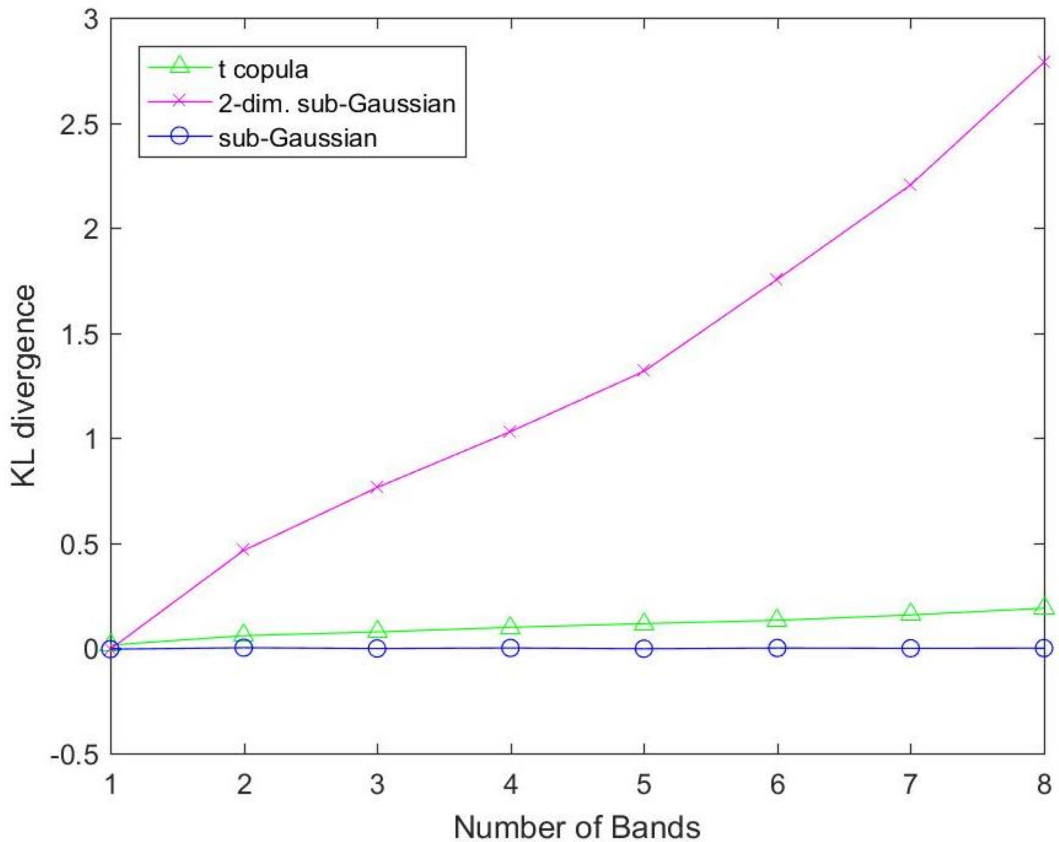
Simulation 4. only stacking real parts, K=8



$$\vec{Z}_K = \left(z_{1,r}, \dots, z_{K,r}\right)$$

Figure 5. KL divergence of $\overrightarrow{Z_K} = (z_{1,r}, \dots, z_{K,r})$ under different p, K=8

Simulation 5. $\vec{Z}_{2K}=\left(z_{1,r},z_{1,i},\ldots,z_{K,r},z_{K,i}\right)$ with *p*=1



Sub-Gaussian

Figure 6. KL divergence of $\overrightarrow{Z_{2K}}=(z_{1,r},\ldots,z_{2K,r})$ under different K, *p*=1

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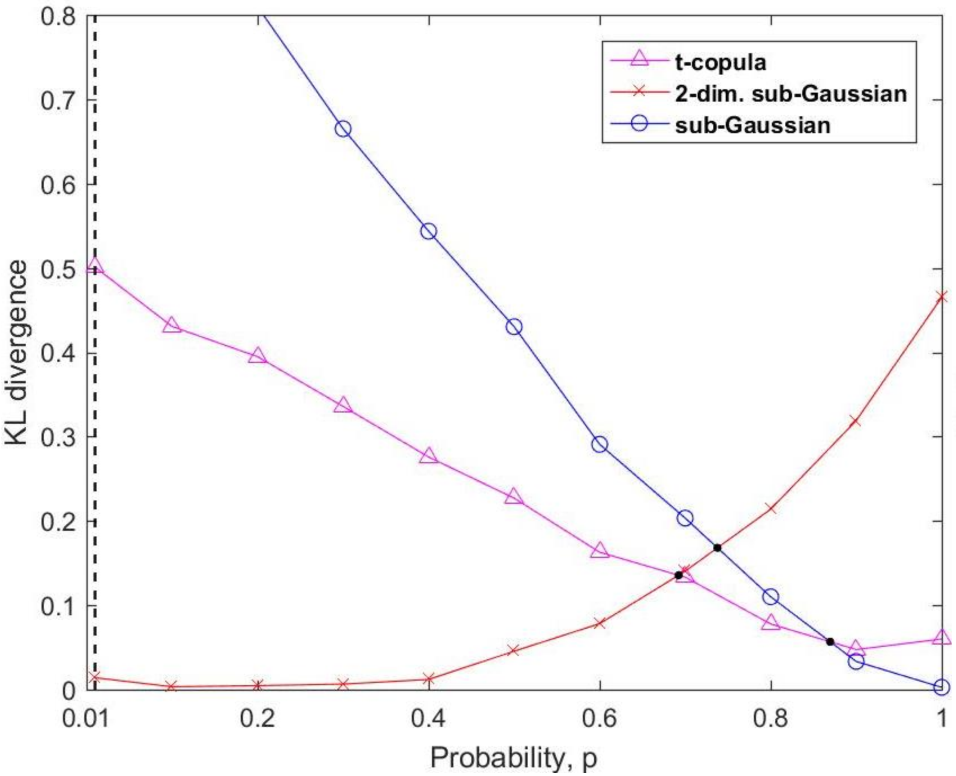
Simulation 6. $\vec{Z}_{2K} = \left(z_{1,r}, z_{1,i}, \ldots, z_{K,r}, z_{K,i} \right)$,K=2 and 8



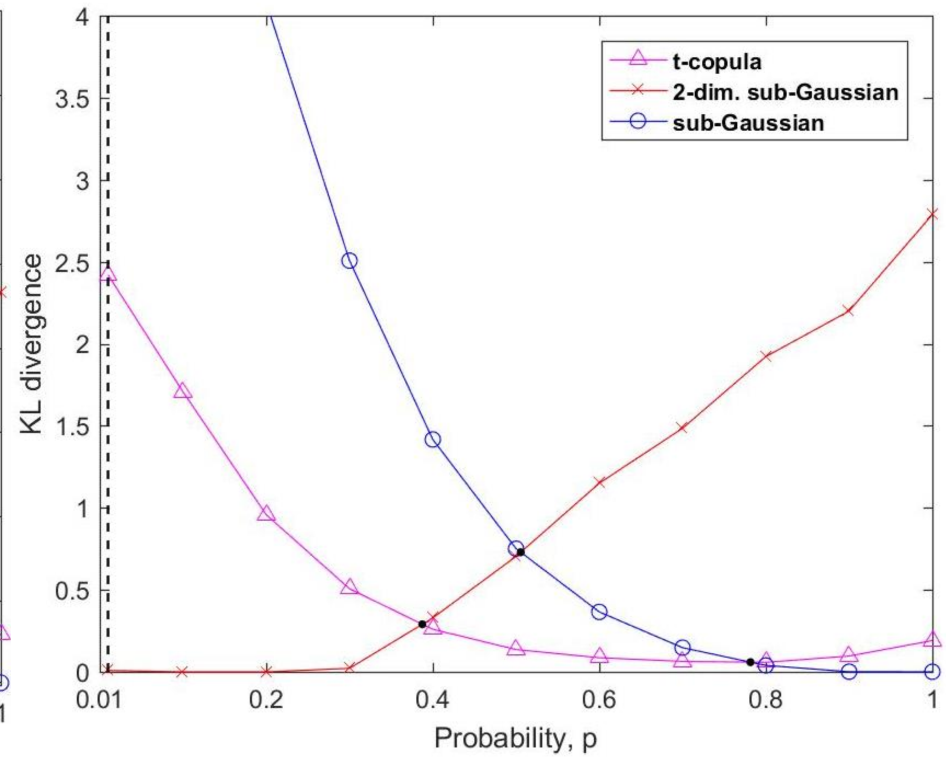


Figure 7. KL divergence of $\overrightarrow{Z_{2K}} = (z_{1,r}, \ldots, z_{2K,r})$ under different *p*, K=2 and K=8

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