Stochastic Resource Allocation for Outage Minimization in Random Access with Correlated Activation

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Background

In industrial fault detection and monitoring,

1. communications are event triggered
2. stochastic process $\rightarrow$ random access policy

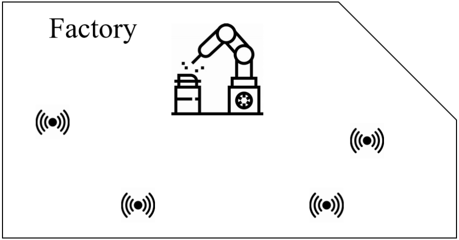

Figure 1: Monitoring of sensor network in a factory

Background

In industrial fault detection and monitoring,

Resource allocation: Aloha assumes independent sensor activity

However, sensor activity are correlated...

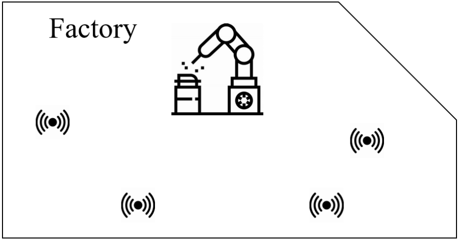

Figure 1: Monitoring of sensor network in a factory

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System Model

In each frame, we have

- K time slots
- N sensors

Each sensor can access only one slot at most.

Activity vector: $\mathbf{X} = \{X_1, \cdots, X_N\} \in \{0, 1\}^N$, where sensor i is active if $X_i = 1$.

1. $X_i, \cdots, X_N$ are dependent (each element)
2. $\mathbf{X^1}, \cdots, \mathbf{X^L}$ are also dependent (each frame)

System Model

In each frame, we have

- K time slots
- N sensors

Each sensor can access only one slot at most.

Activity vector: $\mathbf{X} = \{X_1, \cdots, X_N\} \in \{0, 1\}^N$, where sensor i is active if $X_i = 1$.

Mission: how to allocate slots to sensors when $N > K$?

System Model

How to allocate K slots to N users when $N > K$, with $\mathbf{X}^1, \mathbf{X}^2, \ldots$?

We need to find the allocation matrix $\mathbf{A} \in \mathbb{R}^{N \times K}$ and $\sum_{j=1}^{K} A_{ij} = 1$ for all $i = 1, \cdots, N$. A_{ij} is the probability that user i access slot j conditioned on activation.

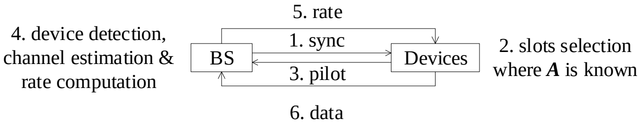
E.g., $N = 3$ users, $K = 2$ slots

$$\mathbf{A} = \begin{bmatrix} \mathbf{0.6} & \mathbf{0.4} \\ 0.1 & 0.9 \\ 0.2 & 0.8 \end{bmatrix}$$

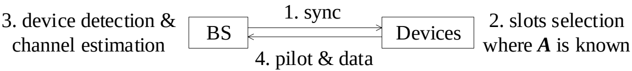
User 1 accesses 1st slot with probability 0.6 and 2nd slot with probability 0.4.

Optimization of allocation matrix $\mathbf{A}$

Protocols



(a) Protocol 1



(b) Protocol 2

Figure 2: Comparison of protocols.

Difference: In Protocol 2 sensors transmit at fixed rate while Protocol 1 has rate adaptation

System Model

At the receiver, successive interference cancellation (SIC) is assumed to be exploited. And the received signal at slot j is

$$\mathbf{y}_j = \mathbf{D}_j \mathbf{g} + \mathbf{w}_j,$$

where $D_j = \pm 1$, $g_i = \sqrt{P} r_i^{-\eta/2} h_i, \ i \in \{1, \ldots, N\}$, $\mathbf{w}_j \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{T \times T})$

- D_j — transmitted data,
- $r_i^{-\eta/2}$ — path loss,
- h_i — fading,
- $\mathbf{w}_j$ — noise

System Model

At the receiver, successive interference cancellation (SIC) is assumed to be exploited. And the received signal at slot j is

$$\mathbf{y}_j = \mathbf{D}_j\mathbf{g} + \mathbf{w}_j,$$

Denote

- $\mathcal{S}_j$ — the set of active sensors in slot j

Consider sensor $m \in \mathcal{S}_j$,

- $|g_m|^2$ — the m-th largest channel gain in $\{|g_i|^2\}_{i\in\mathcal{S}_j}$

The achievable rate (for sensor m in slot j) is

$$R_{j,m} = W \log\left(1 + \frac{|g_m|^2}{\sigma^2 + \sum_{m<l\leq|\mathcal{S}_j|}|g_l|^2}\right), m = 1, \ldots, |\mathcal{S}_j|,$$

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Optimization of $\mathbf{A}$– Criteria

Optimization of allocation matrix $\mathbf{A}$

Find $\mathbf{A}$ that

$$\max \;\; T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

$T^{\mathbf{X}}(\mathbf{A})$, referred as throughput, can be

- $\sum \mathbf{1}$ (*no collision*) — number of slots without collision

$$T^{\mathbf{X}}(\mathbf{A}) = \sum_{n=1}^{N} \sum_{k=1}^{K} X_n A_{nk} \prod_{\substack{m=1 \\ m \neq n}}^{N} (1 - X_m A_{mk}).$$

Optimization of **A**– Criteria

Optimization of allocation matrix **A**

Find **A** that

$$\max \ T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

$T^{\mathbf{X}}(\mathbf{A})$, referred as throughput, can be

- $\sum R_i$, where $R_i = W \log(1 + SINR_i)$ — sum-rate

$$T_3^{\mathbf{h},\mathbf{X}}(\mathbf{A}) = \sum_{k=1}^{K} \sum_{\mathcal{S} \in \mathcal{P}(N)} Q_k(\mathcal{S}|\mathbf{X}) \cdot \sum_{m=1}^{|\mathcal{S}|} \log\left(1 + \frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2 + \sum_{m<l\le|\mathcal{S}|} |g_{\mathcal{S}(l)}|^2}\right)$$

where

$$Q_k(\mathcal{S}|\mathbf{X}) = \prod_{i \in \mathcal{S}} X_i A_{ik} \prod_{j \in \mathcal{S}^c} (1 - X_j A_{jk}),$$

$\mathcal{P}(N)$ denotes the power set of $\{1, \ldots, N\}$ and $\mathbf{1}\{\cdot\}$

Optimization of **A**– Criteria

Optimization of allocation matrix **A**

Find **A** that

$$\max \ T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

$T^{\mathbf{X}}(\mathbf{A})$, referred as throughput, can be

- $\sum \mathbf{1} \ (SINR_i > \theta)$ — number of successful transmission $(R_i > R)$.

$$T^{\mathbf{X}}(\mathbf{A}) = \sum_{k=1}^{K} \sum_{\mathcal{S} \in \mathcal{P}(N)} Q_k(\mathcal{S}|\mathbf{X}) \cdot \sum_{m=1}^{|\mathcal{S}|} \mathbf{1} \left\{ \log \left(1 + \frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2 + \sum_{m < l \le |\mathcal{S}|} |g_{\mathcal{S}(l)}|^2} \right) > R \right\}$$

where

$$Q_k(\mathcal{S}|\mathbf{X}) = \prod_{i \in \mathcal{S}} X_i A_{ik} \prod_{j \in \mathcal{S}^c} (1 - X_j A_{jk}),$$

$\mathcal{P}(N)$ denotes the power set of $\{1, \ldots, N\}$ and $\mathbf{1}\{\cdot\}$

Optimization of **A**– Criteria

Optimization of allocation matrix **A**

Find **A** that

$$\max \ T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

$T^{\mathbf{X}}(\mathbf{A})$, referred as throughput, can be

- $\sum \mathbf{1}$ $(\textit{no collision})$ — number of slots without collision;
- $\sum R_i$, where $R_i = W \log(1 + SINR_i)$ — sum-rate;
- $\sum \mathbf{1}$ $(SINR_i > \theta)$ — number of successful transmission $(R_i > R)$.

Therefore, three algorithms based on Stochastic Gradient Ascent (SGA) corresponding each are considered.

Stochastic Gradient Ascent

$$\max \quad T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

$\mathbf{A}$ is what we want to optimized.

The Algorithm is given as follows:

1: **Input:** Choose an initial iterate $\mathbf{A}^1$ and step-size $\alpha^1 > 0$.

2: $t \leftarrow 0$.

3: **While** not converged

4: Using $\mathbf{X}^t$, compute an unbiased estimate $\mathbf{Y}^t(\mathbf{A}^t)$ of $\nabla \mathrm{T}\ (\mathbf{A}^t)$

5: Set $\mathbf{A}^{t+1} \leftarrow \Pi_{\mathcal{H}}[\mathbf{A}^t + \alpha^t \mathbf{Y}^t(\mathbf{A}^t)]$.

6: $t \leftarrow t + 1$.

7: End **While**

8: **Output:** $\mathbf{A}^t$.

Stochastic Gradient Ascent

$$\max \ T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

A is what we want to optimized.

The performance of Stochastic gradient ascent is mainly decided by

1. the learning rate or step size;
2. **the initial value**.

Choice of Initial Value

Therefore, we consider the initial value of $\mathbf{A}$ as

$$\mathbf{A}_1 = \begin{bmatrix} 1/K & 1/K & \dots & 1/K \\ 1/K & 1/K & \dots & 1/K \\ \vdots & \vdots & \ddots & \vdots \\ 1/K & 1/K & \dots & 1/K \end{bmatrix}_{N\times K}$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}_{N\times K}, \qquad \mathbf{A}_3 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & & 1 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}_{N\times K}$$

and $\mathbf{A}_4$ and $\mathbf{A}_5$, which correspond to the allocations obtained via $\mathbf{Alg_{upper}}$ and $\mathbf{Alg_{lower}}$ from Anders [1]

[1] Anders E Kalor, et al, "Random access schemes in wireless systems with correlated user activity", In SPAWC 2018

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Simulation

We consider $N = 10$ sensors in the network, uniformly distributed on the circles with distances from the access point given by

$$\begin{aligned}\mathbf{d} =& [0.1743, 0.8433, 1.2320, 1.5303, 5.8221,\\ & 6.5653, 7.1598, 8.0840, 8.4006, 9.7600].\end{aligned}$$

Recall that

$$\max \ T(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T^{\mathbf{X}}(\mathbf{A})],$$

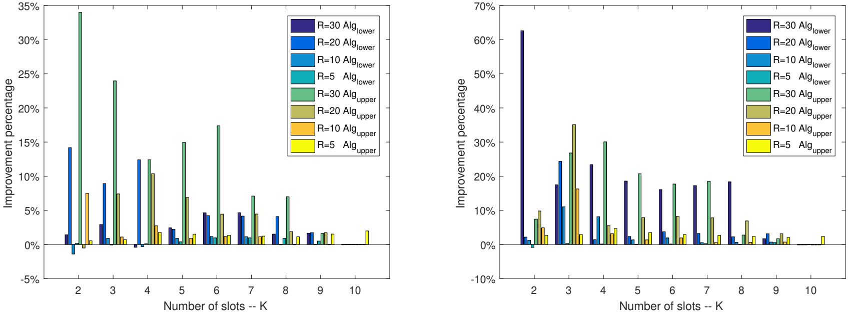
- $\sum \mathbf{1}$ (*no collision*) — $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$;
- $\sum R_i$ based on sum-rate — **Algorithm 3**;
- $\sum \mathbf{1}$ $(SINR_i > \theta)$ based on number of successful transmission — **Algorithm 4**.

SINR $\tau = e^R - 1$

Compare **Algorithm 4** over **Algorithm 3** or $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$

Simulation

SINR $\tau = e^R - 1$



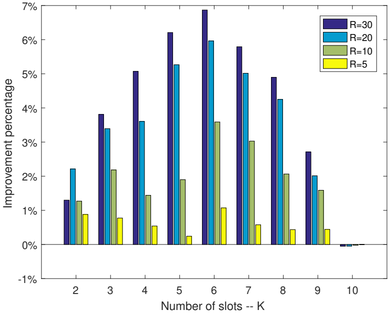
(a) $\tau = 0$ dB (b) $\tau = 20$ dB

Figure 3: Improvement of Algorithm 4 over $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$ with $N = 10$ sensors for varying K for different τ

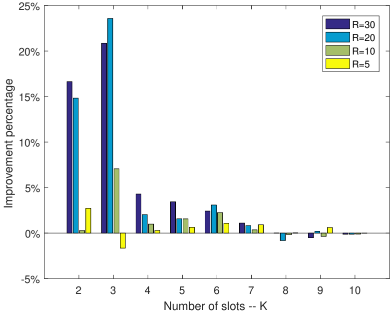
- $\sum \mathbf{1}$ (*no collision*) — $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$;
- $\sum \mathbf{1}$ $(SINR_i > \theta)$ based on number of successful transmission — **Algorithm 4**.

Simulation

SINR $\tau = e^R - 1$



(a) $\tau = 0$ dB



(b) $\tau = 20$ dB

Figure 4: Improvement of Algorithm 4 over Algorithm 3 with $N = 10$ sensors for varying K for different τ

- $\sum R_i$ based on sum-rate — **Algorithm 3**;
- $\sum \mathbf{1}\ (SINR_i > \theta)$ based on number of successful transmission — **Algorithm 4**.

Conclusion

A key problem in the design of random access communications for event-triggered sensor networks is minimizing outages in the presence of correlated activation.

In this paper, we developed a new stochastic resource optimization algorithm, which can outperform existing methods when a limited number of slots are available per frame.

A further benefit of our approach is that parameters of the distribution governing activation do not in general need to be directly estimated.

References

- **Ce Zheng**, Malcolm Egan, Laurent Clavier, Petar Popovski, Anders Ellersgaard Kalør, “Stochastic Resource Optimization of Random Access for Transmitters with Correlated Activation”, *IEEE Communication Letters*, 2021
- Anders E Kalor, Osama A Hanna, Petar Popovski, “Random access schemes in wireless systems with correlated user activity”, In2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)
- **Ce Zheng**, Malcolm Egan, Laurent Clavier, Petar Popovski, Anders Ellersgaard Kalør, “Stochastic Resource Allocation for Outage Minimization in Random Access with Correlated Activation” In Proc. Global Communications Conference (WCNC) 2022