Linear Combining in Dependent α-Stable Interference

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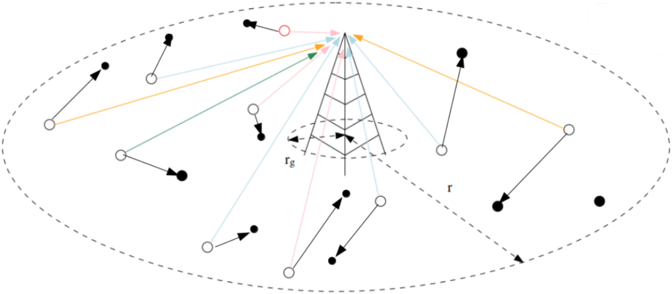
Interference in the IoT

A key challenge in the IoT is to manage interference from a large number of uncoordinated devices.



Many devices operate with only duty cycle constraints $\Rightarrow$ **as the number of devices increases so does the interference problem.**

A fundamental problem is to identify the interference statistics and develop signal processing strategies to minimize performance loss.

Interference in the IoT

Stochastic geometry is a popular technique to model interferer locations.

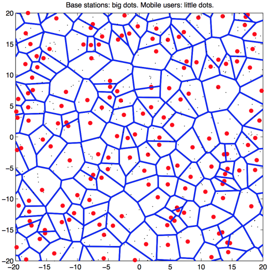


Figure 1: A Poisson spatial field of interferers [Andrews2011].

For a homogeneous Poisson point process model, *scalar* interference is well approximated by a **non-Gaussian α-stable model** [Sousa1992, Ilow1998, Pinto2010, Gulati2010].

Other point process models (e.g., Poisson-Poisson cluser processes) can also induce α-stable interference [Zheng2020].

Interference in the IoT

Recently, NB-IoT has become a popular standard for IoT networks.

NB-IoT is based on OFDM, allowing for multi-tone transmissions.

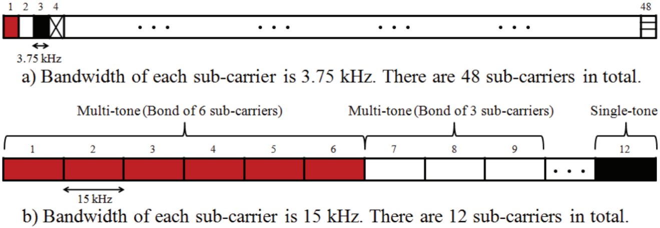


Figure 2: NB-IoT transmission modes: (a) single-tone mode; (b) single-tone and multi-tone mode [Mostafa2017].

Devices exploiting NB-IoT induce *vector* interference corresponding to each tone.

Interference in the IoT

There are two basic questions that we will address:

(1) **What are the statistics of vector interference induced by multi-tone transmissions?**

For devices located according to a Poisson point process, the interference for each tone (or subband) is α-stable. However, the dependence between interference on each tone is not well understood.

(2) **How should a receiver perform detection in the presence of NB-IoT interference?**

System Model

Consider a network of IoT interferers that each transmit on a subset on the set of subbands $\{1, \ldots, K\}$. The set of all interferers is assumed to form a homogeneous Poisson point process.

The interference on the i-th subband is given by

$$z_i = \sum_{j \in \Phi_i} r_j^{-\eta/2} x_{j,i}, \quad i \in \{1, \cdots, K\}$$

where

- Φ_i is the set of interferers on subband i;
- r_j is distance from device j in Φ_i to the desired receiver;
- η is the path loss;
- $x_{j,i} \in \mathbb{R} \sim \mathcal{N}(0, \sigma_I^2)$ is the combination of baseband emission and small-scale fading from interferer j on subband i..

System Model

Under the system model, if all devices transmit on subband i, the interference is α-stable [Egan2018].

Theorem 1

Suppose that

$$\mathbb{E}[|x_{j,i}|^{4/\eta}] < \infty,$$

with $\eta > 2$. *Then,* z_i *converges almost surely to a symmetric* $4/\eta$*-stable random variable with the scale parameters given by*

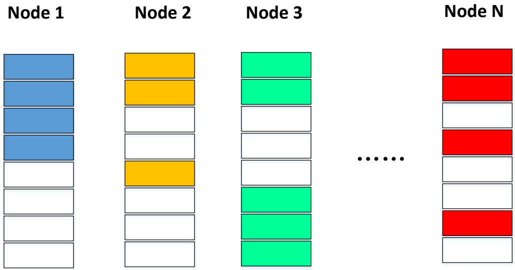
$$\gamma_N = \left(\pi \lambda p C_{\frac{4}{\eta}}^{-1} \mathbb{E}[|x_{j,i}|^{\frac{4}{\eta}}]\right)^{\frac{\eta}{4}},$$

That is, z_i has characteristic function $\Phi_{z_i}(t) = \exp(-\gamma_N^{4/\eta}|t|^{4/\eta})$.

System Model

In order to characterize the statistics of the interference z_i, it is necessary to specify Φ_i.

The devices are assumed to be uncoordinated and access each subband independently with probability p.



Interference Statistics

We now address our first question:

(1) **What are the statistics of vector interference induced by multi-tone transmissions?**

In the special case where $p = 1$, where all devices transmit on each tone, the interference vector $\mathbf{z}$ is **sub-Gaussian** α**-stable** [Zheng2020].

Interference Statistics

Definition 2

Any vector $\mathbf{X}$ distributed as $\mathbf{X} = [A^{1/2}G_1, \cdots, A^{1/2}G_d]^T$ is called a sub-Gaussian α-stable random vector in $\mathbb{R}^d$ with underlying Gaussian vector $\mathbf{G} = [G_1, \ldots, G_d]^T$ if it satisfies

$$A \sim S_{\alpha/2}\left(\left(cos\frac{\pi}{4}\alpha\right)^{2/\alpha}, 1, 0\right), \tag{1}$$

where A and $\mathbf{G}$ are independent, and $\mathbf{G} \sim \mathcal{N}(0, \sigma^2\mathbf{I})$.

It is conditionally Gaussian $\mathbf{X} \sim \mathcal{N}(0, \sigma^2 A\mathbf{I})$.
Its characteristic function is:

$$\mathbb{E}[e^{i\boldsymbol{\theta}\cdot\mathbf{z}}] = \exp\{\gamma^\alpha|\boldsymbol{\theta}|^\alpha\}$$

However, the statistics for general p are more complicated. We focus on the case of $K = 2$ tones.

Interference Statistics

Theorem 3

$(0 < p < 1)$ For $K = 2$, the characteristic function of (z_1, z_2) is of the form

$$\Phi_{\mathbf{z}}(\boldsymbol{\theta}) = \mathbb{E}[e^{i(\theta_1 z_1 + \theta_2 z_2)}] = \exp\{\underbrace{i\gamma_1^\alpha |\theta_1^2 + \theta_2^2|^{\frac{\alpha}{2}}}_{\Gamma_1} + \underbrace{i\gamma_2^\alpha (|\theta_1|^\alpha + |\theta_2|^\alpha)}_{\Gamma_2}\}$$

where

$$\gamma_1 = \sigma_I \left(\pi\lambda p^2 C_{4/\eta}^{-1} \mathbb{E}[|Z_0|^{4/\eta}]\right)^{\eta/4}$$
$$\gamma_2 = \sigma_I \left(\pi\lambda p(1-p) C_{4/\eta}^{-1} \mathbb{E}[|Z_0|^{4/\eta}]\right)^{\eta/4}$$

That is, $\mathbf{z}$ is a symmetric α-stable random vector with spectral measure on $\mathbb{S}^1$ given by $\Gamma = \Gamma_1 + \Gamma_2$, with Γ_1 uniform on $\mathbb{S}^1$ and Γ_2 concentrated on $(\pm 1, 0), (0, \pm 1)$.

Interference Statistics

$$\Phi_{\mathbf{z}}(\boldsymbol{\theta}) = \mathbb{E}[e^{i(\theta_1 z_1 + \theta_2 z_2)}] = \exp\{\underbrace{i\gamma_1^\alpha|\theta_1^2 + \theta_2^2|^{\frac{\alpha}{2}}}_{\Gamma_1} + \underbrace{i\gamma_2^\alpha(|\theta_1|^\alpha + |\theta_2|^\alpha)}_{\Gamma_2}\}$$

Recall that the α-stable distribution has the characteristic function

$$\mathbb{E}[e^{i\theta \cdot z}] = \exp\{i\gamma^\alpha|\theta|^\alpha\}$$

and the sub-Gaussian α-stable has

$$\mathbb{E}[e^{i\boldsymbol{\theta}\cdot\mathbf{z}}] = \exp\{\gamma^\alpha|\boldsymbol{\theta}|^\alpha\}$$

Hence, the general interference is a mixing of sub-Gaussian α-stable (Γ_1) and independent α-stable (Γ_2).

Receiver Design

Consider a device seeking to transmit data to an intended receiver in the presence of IoT interference.

The received signal is

$$\mathbf{y} = \mathbf{h}x + \mathbf{z},$$

where

- $\mathbf{h} \in \mathbb{R}^K$ is the channel fading;
- $x \in \{+1, -1\}$ is the transmitted binary symbol;
- $\mathbf{z} \in \mathbb{R}^K$ is the IoT interference vector.

(2) **How should a receiver perform detection in the presence of NB-IoT interference?**

System Model

The optimal receiver to minimize the BER exploits the likelihood ratio

$$\Lambda(\mathbf{y}) = \frac{f(\mathbf{y}|x=1)}{f(\mathbf{y}|x=-1)} \underset{x=-1}{\overset{x=1}{\gtrless}} 1.$$

However, the interference is based on an α-stable model which does not admit a closed-form probability density function.

Instead, we consider the optimal *linear* combining; i.e.,

$$\tilde{y} = \mathbf{w}^T \mathbf{y} \underset{x=-1}{\overset{x=1}{\gtrless}} 0.$$

where $\mathbf{w} \in \mathbb{R}^K$ and $\|\mathbf{w}\| = 1$.

Optimal Linear Combining for $p = 1$

When $\mathbf{z}$ is sub-Gaussian α-stable (i.e., $p = 1$) the **optimal linear combining is maximum ratio combining (MRC)**, i.e., $\mathbf{w} = \mathbf{h}/\|\mathbf{h}\|$.

The claim follows from an argument based on [Niranjayan2009] for independent α-stable interference.

The optimal linear receiver for $p = 1$ **is the same as for Gaussian interference**.

Optimal Linear Combining for $p = 1$

The BER admits an expression that can be viewed in terms of a *fractional diversity gain*

Theorem 4

Let $\mathbf{z}$ *be a* K*-dimensional sub-Gaussian* α*-stable random vector with parameter* $\sigma_{\mathbf{z}}$ *and the linear combining weights be* $\mathbf{w} \in \mathbb{R}^K$*. Then, as* $\|\mathbf{h}\| \to \infty$,

$$P_e(\mathbf{w}) = \frac{1}{2} C_\alpha \gamma_{\mathbf{z}}^{\alpha} \left(\mathbf{w}^T \mathbf{h} / \|\mathbf{w}\| \right)^{-\alpha} + o\left(\left(\mathbf{w}^T \mathbf{h} / \|\mathbf{w}\| \right)^{-\alpha} \right).$$

where $\gamma_{\mathbf{z}} = \sigma_{\mathbf{z}} / \sqrt{2}$*.*
Moreover, the optimal linear weights ($\mathbf{w} = \mathbf{h} / \|\mathbf{h}\|$*) admit a BER*

$$P_e(\mathbf{h}) = \frac{1}{2} C_\alpha \gamma_{\mathbf{z}}^{\alpha} \|\mathbf{h}\|^{-\alpha} + o(\|\mathbf{h}\|^{-\alpha}).$$

Optimal Linear Combining with $p = 1$

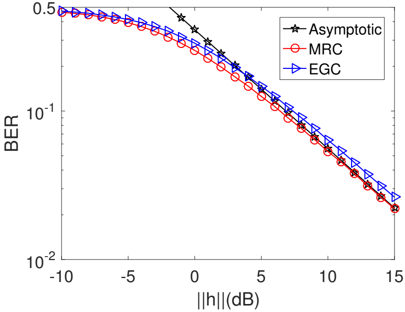


Figure 4: Bit error rates comparison of MRC and EGC for sub-Gaussian α-stable interference ($p = 1$) under different $\|h\|$ with $K = 10$ channels, $\gamma_{\mathbf{z}} = 1$, $\alpha = 4/5$ and $x = \pm 1$

- Asymptotic: $P_e(\mathbf{h}) = \frac{1}{2} C_\alpha \gamma_{\mathbf{z}}^\alpha \|\mathbf{h}\|^{-\alpha} + o(\|\mathbf{h}\|^{-\alpha})$
- MRC: Maximum ratio combining $\mathbf{w} = \mathbf{h}/||\mathbf{h}||$
- EGC: Equal gain combining $\mathbf{w} = \frac{1}{K}\mathbf{1}_K$

Optimal Linear Combining with $p = 1$

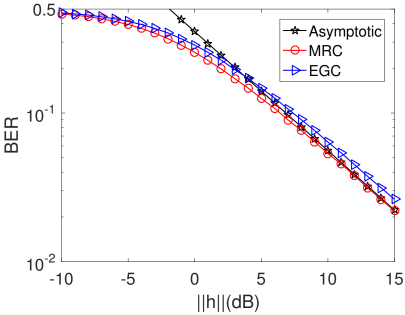


Figure 4: Bit error rates comparison of MRC and EGC for sub-Gaussian α-stable interference ($p = 1$) under different $\|h\|$ with $K = 10$ channels, $\gamma_{\mathbf{z}} = 1$, $\alpha = 4/5$ and $x = \pm 1$

- The MRC combiner outperforms EGC especially for large value of $||\mathbf{h}||$
- The asymptotic approximation for the BER is in good aggreement with Monte Carlo simulation for large $||\mathbf{h}||$

Optimal Linear Combining with $0 < p < 1$

The optimal linear receiver for $p = 1$ is the same as for Gaussian interference.

This is not true for general $0 < p < 1$.

Theorem 5

Let $\mathbf{z}$ *have the characteristic function given in Theorem 3, corresponding to the general interference model. Then, the optimal linear combining (OLC) weights are the solution of*

$$\max_{\mathbf{w}\in\mathbb{R}^2:\|\mathbf{w}\|=1} \frac{\mathbf{w}^T\mathbf{h}}{\left(\gamma_1^\alpha(w_1^2+w_2^2)^{\alpha/2}+\gamma_2^\alpha|w_1|^\alpha+\gamma_2^\alpha|w_2|^\alpha\right)^{1/\alpha}}$$

Optimal Linear Combining with $0 < p < 1$

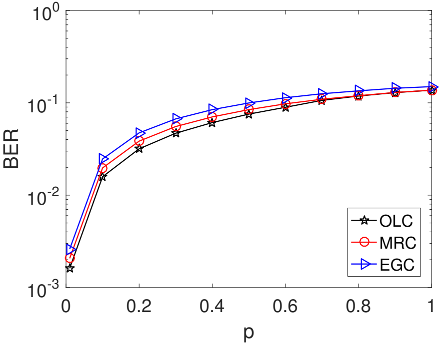
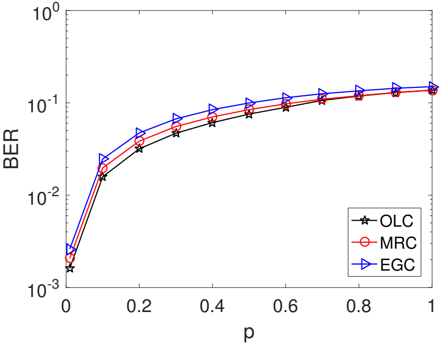


Figure 5: Bit error rates of OLC, MRC and EGC under different p with $\lambda = 0.001\ \mathrm{m}^{-2}$, $\|\mathbf{h}\| = 10^{-3}$, $\eta = 5$, and $\sigma_I = 1$.

$p\uparrow$:
more devices transmitting, the scale parameter for marginals $\uparrow$
$\Rightarrow$
increase the BER

$$\gamma_N = \left(\pi\lambda p C_{\frac{4}{\eta}}^{-1}\mathbb{E}[|x_{j,i}|^{\frac{4}{\eta}}]\right)^{\frac{\eta}{4}},$$

Optimal Linear Combining with $0 < p < 1$



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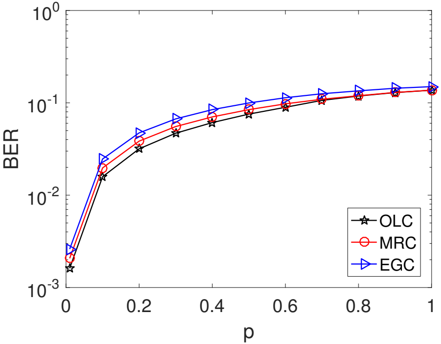
$p\uparrow$:
$\mathbf{z}$ more dependent and approximately sub-Gaussian for $p \approx 1$
$\Rightarrow$
reduce the BER

Optimal Linear Combining with $0 < p < 1$



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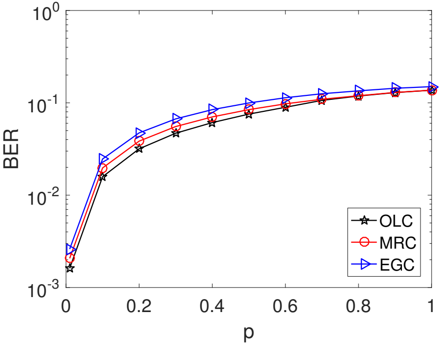
Example:
$w_1 z_1 + w_2 z_2$, $w_1 = w_2 = 1$ and $z_i \sim S_\alpha(1, 0, 0)$, $\gamma_{\mathbf{w}} = \sqrt{2}$ for sub-Gaussian and $\gamma_{\mathbf{w}} = 2^{1/\alpha}$ for independent subbands ($\alpha < 2$).

Optimal Linear Combining with $0 < p < 1$



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The impact on the BER of the scale parameter dominates the impact of dependence as p increases.

Conclusion

1. Prove that MRC is the optimal linear combining for sub-Gaussian α-stable
2. Derive an asymptotic approximation of the BER
3. For general scenario ($0 < p < 1$), establish an exact characterization of the interference random vector and the optimal weights for linear combining.

References

[Zheng2019] Zheng, C., Egan, M., Clavier, L., Peters, G.W. and Gorce, J.-M., "*Copula-Based Interference Models for IoT Wireless Networks*", IEEE International Conference on Communications (ICC), 2019.

[Zheng2020] Zheng, C., Egan, M., Clavier, L., Peters, G.W. and Gorce, J.-M., "*Impulsive Multivariate Interference Models for IoT Networks*", available:
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