Interference Modeling for Wireless IoT Networks

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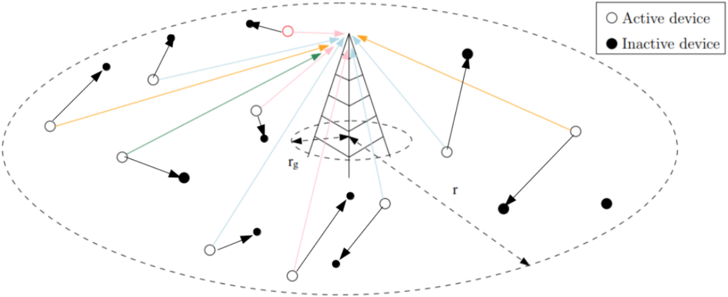
Communication for the IoT

In the Internet of Things, huge numbers of devices transmitting to different access points must coexist.



The Interference Problem

The IoT is expected to operate in the ISM bands:

- Low power wide area networks (e.g., SigFox and LoRa) on 863-870 MHz bands.
- ZigBee
- Radio frequency identification (RFID)
- Various devices (e.g., alarms, car keys, etc)

Most devices are uncoordinated!

The Interference Problem

ETSI and ERC recommendations for ISM bands require that transmitting devices either:

- Listen before talk (listen > 5 s)

Bluetooth/Zigbee – IEEE 802.15 type devices

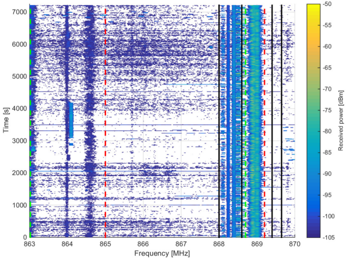
- Restrict duty cycles (maximum percentage of on time per hour)

LoRa and SigFox rely on duty cycle access

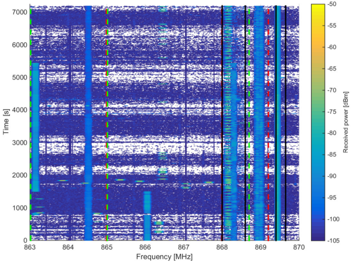
$\Rightarrow$

Increasing interference with an increasing number of devices.

The Interference Problem



(a) Shopping area.



(b) Business area.

Figure 1: Interference experimental results in Aalborg from [Lauridsen, 2017].

The Interference Problem

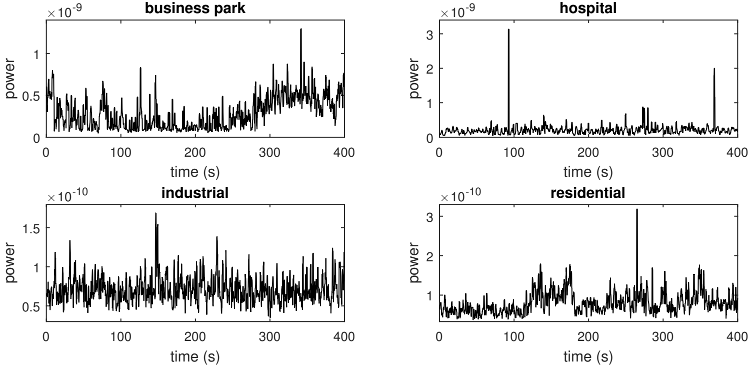


Figure 2: Example of a few interference samples measured in the different areas

The Standard Interference Modeling Approach [Aalborg University]

In many networks, device locations are modeled using point processes.

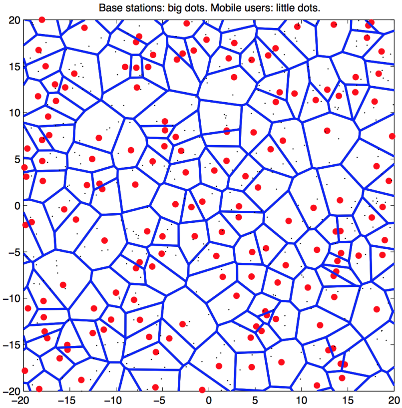
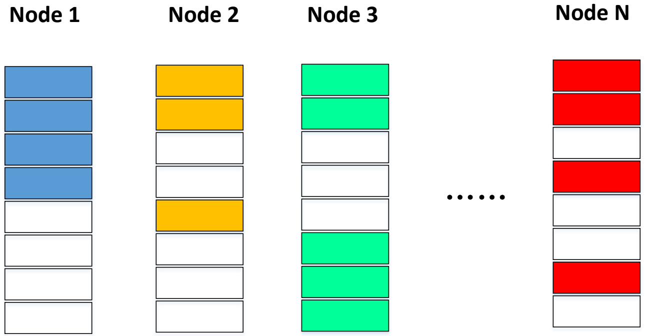


Figure 3: A Poisson spatial field of interferers [Andrews2011].

Access Scheme

- Uncoordinated: interfering devices independently transmit on band $k \in \mathcal{B}$



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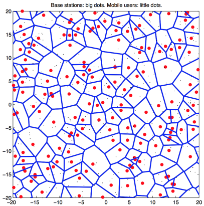
The Standard Interference Modeling Approach



The interference is given by

$$Z = \sum_{i \in \Phi} r_i^{-\eta/2} h_i X_i.$$

Conditioned on Φ and (h_i), Z is then assumed to be Gaussian.

For example, the expected rate is given by

$$\overline{R} = \mathbb{E}_{\Phi, h_i}[\log(1 + \mathrm{SINR})].$$

This only makes sense when the set of interfering devices does not change.

I.e., the devices transmit long packets.

New Challenges in the IoT

In IoT communication networks, devices send small amounts of data.

That is, they send short packets.

$\Rightarrow$ **Interferers can change during a transmission.**

$\Rightarrow$ We cannot condition on the locations Φ to obtain a Gaussian model.

$$Z = \sum_{i \in \Phi} r_i^{-\eta/2} h_i X_i.$$

This scenario is called **dynamic interference**.

Initial Steps

Question: For each received symbol, what are the statistics of the interference?

In scenarios where

- devices are located according to a Poisson point process
- transmission is on a single subcarrier

the interference statistics have been extensively studied [Middleton1977,Sousa1992,Ilow1998,Yang2003,Pinto2010,Gulati2010].

Initial Steps

This earlier work has established the interference statistics on a single subcarrier.

$$Z = \sum_{i \in \Phi} r_i^{-\eta/2} h_i X_i.$$

Suppose that

(i) the point process Φ is homogeneous Poisson;

(ii) (h_i) and (X_i) are independent

(iii) each h_i or X_i is isotropic;

(iv) $\mathbb{E}[|\mathrm{Re}(h_i X_i)|^{4/\eta}] < \infty$.

Then the interference Z is isotropic complex $4/\eta$-stable.

Observation: the interference is non-Gaussian.

α-Stable Models

- A random variable X has a *stable distribution* if for any positive numbers A and B, there is a positive number C and a real number D such that

$$AX_1 + BX_2 \overset{d}{=} CX + D,$$

where X_1 and X_2 are independent copies of X.

- The characteristic function of an α-stable random variable X is given by

$$\mathbb{E}[e^{i\theta X}] = \begin{cases} \exp\left\{-\gamma^\alpha|\theta|^\alpha(1 - i\beta(\mathrm{sign}\theta)\tan\frac{\pi\alpha}{2}) + i\delta\theta\right\}, & 0 < \alpha < 2,\ \alpha \neq 1 \\ \exp\left\{-\gamma|\theta|(1 + i\beta\frac{2}{\pi}(\mathrm{sign}\theta)\log|\theta|) + i\delta\theta\right\}, & \alpha = 1 \end{cases}$$

α – impulsiveness, β – skewness, γ – dispersion, δ – location.

α-Stable Models

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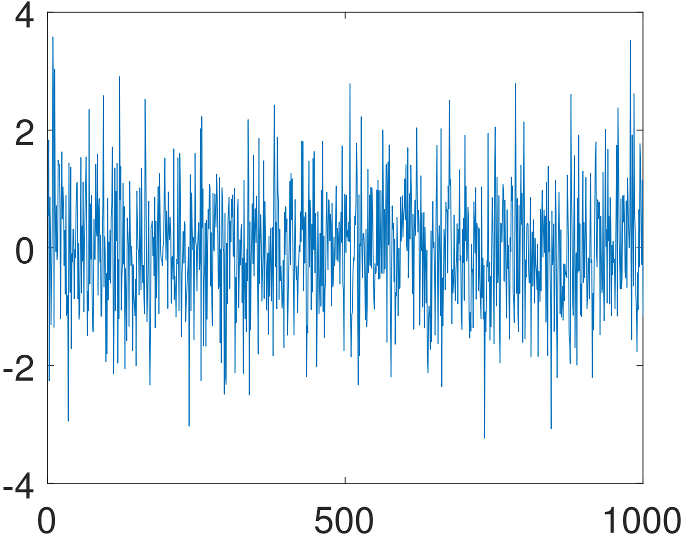
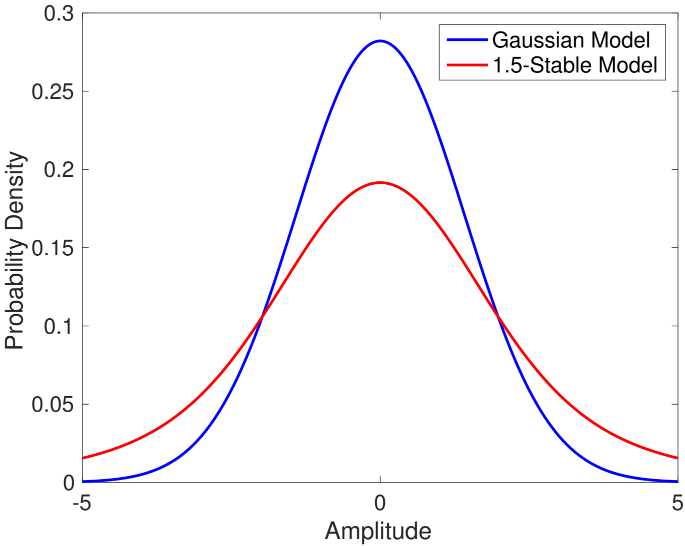
α-Stable Models



α-Stable Models



α-Stable Models

Under the Poisson point process model, the interference on a single subcarrier Z is isotropic complex $4/\eta$-stable.

$$Z = \sum_{i \in \Phi} r_i^{-\eta/2} h_i X_i.$$

This means that

$$\mathbf{Z} = [\mathrm{Re}(Z), \mathrm{Im}(Z)]^T \stackrel{d}{=} A^{1/2}\mathbf{G},$$

where

- A is a skewed α-stable random variable.
- $\mathbf{G} \sim \mathcal{N}(\mathbf{0}_{2\times 1}, \sigma^2 \mathbf{I}_{2\times 2})$

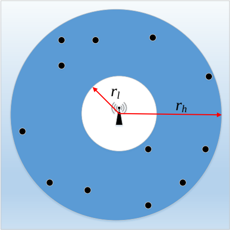
Isotropic α-stable random variables are a special case of **sub-Gaussian α-stable random vectors**.

These will play an important role later.

Interference Approximation in Single Band Systems

In practice,

- devices cannot be arbitrarily close to the receiver
- the network radius is not infinite



Kullback-Leibler Divergence:

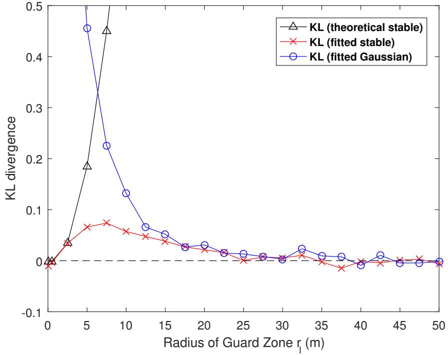
$$D(P_{\mathbf{X}}||P_{\mathbf{Y}}) = \int_{\mathbb{R}^d} p_{\mathbf{X}}(\mathbf{x}) \log \frac{p_{\mathbf{X}}(\mathbf{x})}{p_{\mathbf{Y}}(\mathbf{x})} d\mathbf{x}.$$

Interference Approximation in Single Band Systems

In practice,

- devices cannot be arbitrarily close to the receiver
- the network radius is not infinite

Nevertheless, the α-stable model still forms a good approximation.



Design Implications

Much of communication system design relies heavily on the Gaussian noise and interference assumption.

Since the interference model has changed, so does receiver design and network performance.

For example, detection algorithms [ElGhannudi2010], capacity characterization [Egan2018,deFreitas2017], channel coding schemes [Mestrah2018]).

α-stable interference models can significantly change system design.

Receiver Design

In receiver design, the key problem is to obtain an estimate of the data x based on the observation

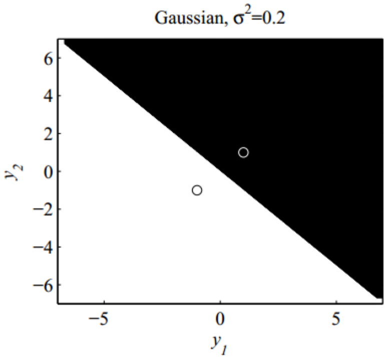
$$Y = x + Z.$$

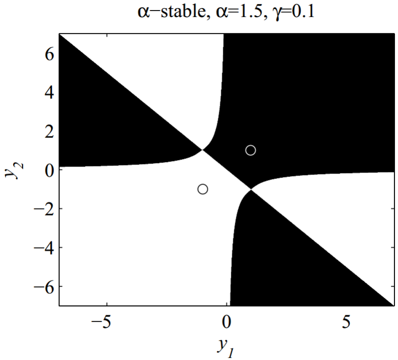
The optimal detector in the Gaussian interference case is based on minimizing the Euclidean distance from the observation to the estimate.

When the interference Z is α-stable, then the decision rule changes.

Receiver Design

In SIMO system, one transmitting anteena and two receiving antennas, with BPSK signals ($x = \pm 1$), the interference vector is $\mathbf{Z} = [Z_1, Z_2]$. Suppose Z_1 and Z_2 are independent symmetric α-stable random variables.





The decision region for BPSK significantly changes (Maximum Likelihood) [Soret, 2017]

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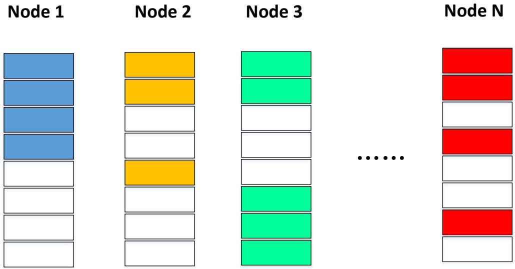
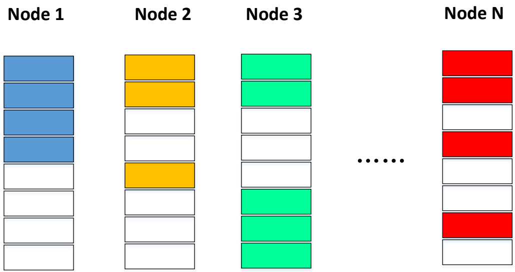
Towards A More Realistic System Setup

- Multi-band scenarios: each device transmits over a subset of orthogonal frequency bands, $\mathcal{B} = \{1, 2, \ldots, K\}$



Towards A More Realistic System Setup

- Multi-band scenarios: each device transmits over a subset of orthogonal frequency bands, $\mathcal{B} = \{1, 2, \ldots, K\}$
- Uncoordinated: interfering devices independently transmit on band $k \in \mathcal{B}$, with probability $p > 0$



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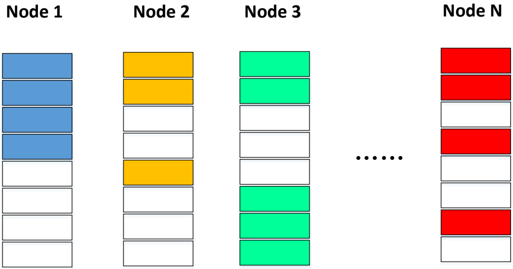
  p – proportional to the quantity of data each interfering device seeks to transmit



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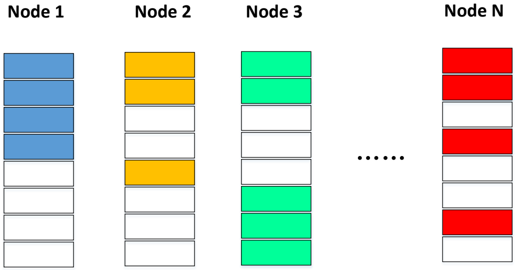
- Probability on M out of K bands — $\binom{K}{M} p^M (1-p)^{K-M}$



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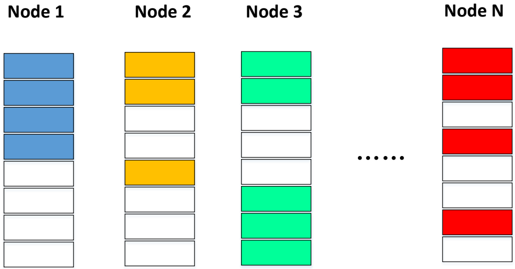
  p – proportional to the quantity of data each interfering device seeks to transmit
- Probability on M out of K bands — $\binom{K}{M} p^M (1-p)^{K-M}$
- Interfering nodes: PPP with $p\lambda$ on the whole plane — Φ



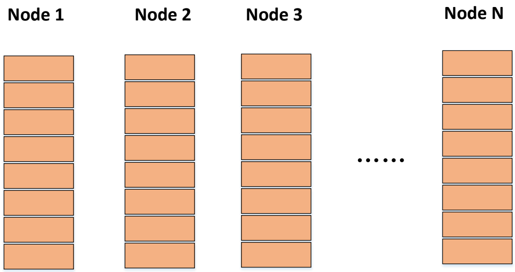
Heavily Loaded Networks: Setup

When all devices transmit on all bands with high probability, the network is said to be **heavily loaded**.

In our model, this corresponds to $p \approx 1$.

This means that the interference is given by

$$Z_k = \sum_{j \in \Phi} r_j^{-\eta/2} h_{j,k} x_{j,k}, \;\; k = 1, \ldots, K.$$



Multivariate α-Stable Models

- A random vector $\mathbf{X} = (X_1, \ldots, X_d)$ is a *stable random vector* in $\mathbb{R}^d$ if for any positive numbers A and B, there is a positive number C and a vector $\mathbf{D} \in \mathbb{R}^d$ such that

$$A\mathbf{X}^{(1)} + B\mathbf{X}^{(2)} \stackrel{d}{=} C\mathbf{X} + \mathbf{D},$$

  where $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are independent copies of $\mathbf{X}$.
- Representation via the characteristic function

$$\Phi_{\mathbf{X}}(\boldsymbol{\theta}) = \mathbb{E}[e^{i\boldsymbol{\theta}\cdot\mathbf{X}}] = \exp\left(-\int_{\mathbb{S}^{d-1}} |\boldsymbol{\theta}\cdot\mathbf{s}|^{\alpha} d\Gamma(\mathbf{s})\right)$$

- **Not all random vectors with α-stable marginals are α-stable.**

Multivariate α-stable Models

*__Sub-Gaussian α-stable distribution__:
$\mathbf{Z} \overset{d}{=} A^{1/2}(G_1, \ldots, G_d)$,

- $A \sim S_{\alpha/2}((\cos \pi\alpha/4)^{2/\alpha}, 1, 0)$ is a skewed α-stable random variable.
- $\mathbf{G} = [G_1, \ldots, G_d]^T \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$

Heavily Loaded Networks: Interference Characterization

The interference random vector is

$$\mathbf{Z} = [\underbrace{\mathrm{Re}(Z_{1,1}), \mathrm{Im}(Z_{1,2})}_{\text{Band 1}}, \cdots, \underbrace{\mathrm{Re}(Z_{K,1}), \mathrm{Im}(Z_{K,N})}_{\text{Band } K}]^T$$

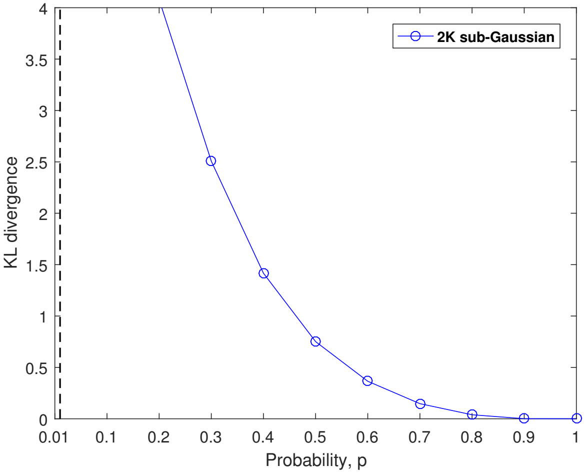
Theorem 1

Suppose that the network is **heavily loaded** *($p = 1$), $h_{j,k}x_{j,k}$ is an isotropic complex random variable for $k = 1, \cdots, K$, and*

$$\mathbb{E}[|\mathrm{Re}(h_{j,k}x_{j,k})|^{4/\eta}|] < \infty, \quad k = 1, \cdots, K$$

Then, $\mathbf{Z}$ *is sub-Gaussian α-stable* with $\alpha = 4/\eta$.*

Heavily Loaded Networks: Interference Characterization



KL divergence between simulated data set and the Sub-Gaussian α-stable model.

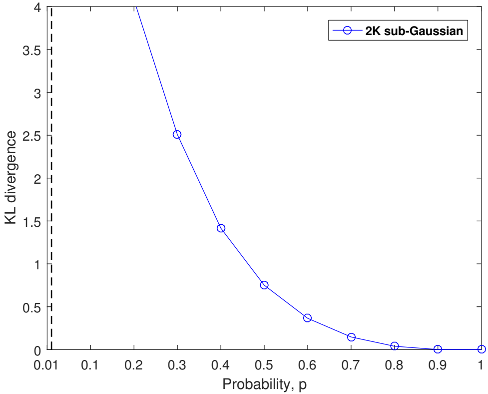
The General Case: Setup

The interference random vector is

$$\mathbf{Z} = [\underbrace{\mathrm{Re}(Z_{1,1}), \mathrm{Im}(Z_{1,2})}_{\text{Band } 1}, \cdots, \underbrace{\mathrm{Re}(Z_{K,1}), \mathrm{Im}(Z_{K,N})}_{\text{Band } K}]^T$$

- Uncoordinated: interfering devices independently transmit on band $k \in \mathcal{B}$, with probability $p > 0$.
- $p = 1$, sub-Gaussian α-stable vector
- $p < 1$, Bands for each node transmitting on partially overlap.

The General Case: Difficulties



Observation: The sub-Gaussian α-stable model does not fit for $p \ll 1$.

Copula Models

Problem: We need an alternative statistical model.

A general way of obtaining multivariate statistical models is via **copula theory**.

For any multivariate distribution, the distribution can be written as:

$$F(x_1, \ldots, x_n) = \prod_{i=1}^{n} F(x_i),$$

$$f(x_1, \ldots, x_n) = \prod_{i=1}^{n} f(x_i),$$

if $x_i, i = 1, \cdots, n$ are independent.

Copula Models

Problem: We need an alternative statistical model.

A general way of obtaining multivariate statistical models is via **copula theory**.

For any multivariate distribution, the distribution can be written as:

$$F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)),$$

where $C : [0,1]^n \to [0,1]$ is called a **Copula** function.
The probability density function isd

$$f(x_1, \dots, x_n) = \underbrace{c(F_1(x_1), \dots, F_n(x_n))}_{\text{Dependence}} \underbrace{\prod_{i=1}^{n} f(x_i)}_{\text{Indepedent}},$$

t-Copula Models

Not all copula models work equally well.

We would also like parameter estimation and simulation to have low complexity.

One option satisfying these criteria is the t-**copula**

$$C^t_{v,\Sigma}(\mathbf{u}) = F_{v,\Sigma}(F_v^{-1}(u_1), \ldots, F_v^{-1}(u_n))),$$

where

- $F_v(x)$ is the CDF of t distribution at x
- and $F_{v,\Sigma}(\mathbf{x})$ is the joint CDF of multivariate t distribution at $\mathbf{x}$.

The General Case: t-Copula Interference Models

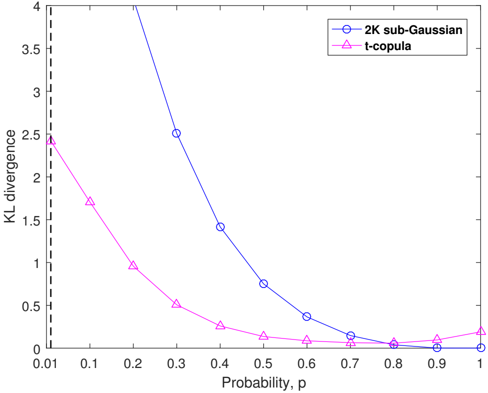
Using the t-copula, we can obtain a new interference model.

Let $H_1, \ldots, H_n$ be the CDFs of stable distributions corresponding to the interference on each subcarrier.

The distribution of $\mathbf{Z}$ is then approximated by the t-**copula interference model**

$$F_{\mathbf{Z}}(\mathbf{x}) \approx C^t_{v,\Sigma}(H_1(x_1), \ldots, H_n(x_n)).$$

The General Case: *t*-Copula Interference Models



Coping with Light Loads

In the **lightly loaded** case ($p \to 0$), the t-copula model does not fit well.

Observation:

- Probability each interfering node transmitting on over two bands ≈ 0.
- No overlaps with high probability.
- By independent thinning theorem for PPPs, interference on any pair of bands is approximately **independent**.

This also explains why the t-copula model does not fit.

Degree of freedom parameter ν in $C^t_{\nu,\Sigma}$ varies significantly between different pairs of bands.

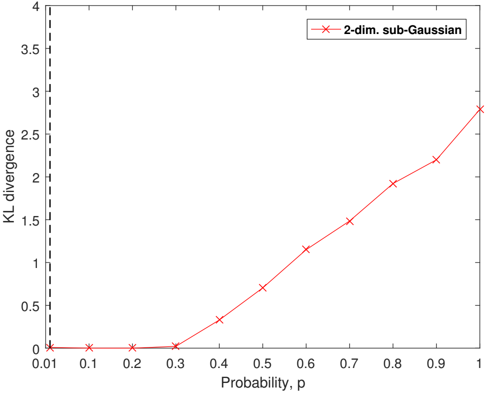
Coping with Light Loads: A New Model

In the lightly loaded scenario ($p \approx 0$) a good approximation is the independent sub-Gaussian α-stable model:

$$\mathbf{Z} = [\underbrace{\mathrm{Re}(Z_{1,1}), \mathrm{Im}(Z_{1,2})}_{\text{Band 1}} \cdots \underbrace{\mathrm{Re}(Z_{K,1}), \mathrm{Im}(Z_{K,2})}_{\text{Band } K}]^T$$

where each band is an independent 2-dimensional sub-Gaussian α-stable random vector.

Coping with Light Loads: Validation



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Summary of the Model

We now have a good characterization of the interference random vector

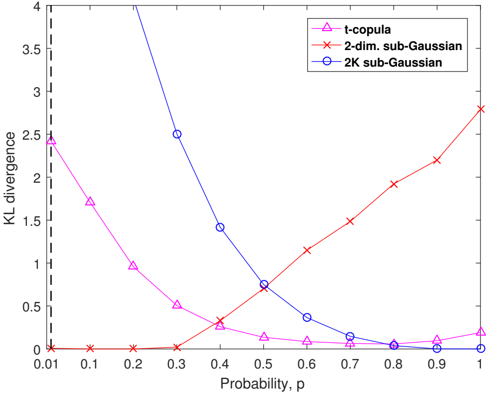
$$\mathbf{Z} = [\underbrace{\mathrm{Re}(Z_{1,1}), \mathrm{Im}(Z_{1,2})}_{\text{Band 1}} \cdots \underbrace{\mathrm{Re}(Z_{K,1}), \mathrm{Im}(Z_{K,2})}_{\text{Band } K}]^T$$

When the network is heavily loaded ($p = 1$) the interference random vector is **sub-Gaussian** α**-stable**.

When the network is moderately loaded ($0 \ll p < 1$), a good approximation is the t**-copula model with** α**-stable marginals**.

When the network is lightly loaded ($p \approx 0$), a good approximation is the **independent model** (each band is sub-Gaussian α-stable).

Summary of the Model



Conclusions

Interference is a key bottleneck in the design of IoT communication systems.

Due to short packets, interferers can change rapidly compared with classical models.

We have studied the interference statistics of a Poisson spatial field of interferers with multiple bands.

By using new tools from copula theory, we have obtained tractable approximations for the interference statistics.

This forms the basis for improved design of IoT networks.

References

[ZEC+19] Zheng, C., Egan, M., Clavier, L., Peters, G.W. and Gorce, J.-M., "*Copula-Based Interference Models for IoT Wireless Networks*", IEEE International Conference on Communications (ICC), 2019.

[ZEC+] Zheng, C., Egan, M., Clavier, L., Peters, G.W. and Gorce, J.-M., "*Statistical Characterization and Estimation for Interference Random Vectors in Poisson Spatial Fields of Interferers*", to be submitted.